HYPER VERSION

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Warped AdS near-horizon geometry of completely localized intersections of M5-branes

Ansar Fayyazuddin

Department of Physics, Stockholm University
Box 6730, S-113 85 Stockholm, Sweden
E-mail: ansar@physto.se

Douglas J. Smith

Department of Mathematical Sciences, University of Durham
Durham, DH1 3LE, UK
E-mail: Douglas.Smith@durham.ac.uk

Abstract: We study a Hanany-Witten set-up relevant to $\mathcal{N} = 2$ superconformal field theories. We find the exact near-horizon solution for this 11-dimensional system which involves intersecting M5-branes. The metric describes a warped product of AdS_5 with a manifold with SU(2) $\times$ U(1) isometry.

Keywords: M-Theory, p-branes, AdS/CFT Correspondance.

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1. Introduction

Intersecting branes are ubiquitous in string theory. Although much effort has been put into finding supergravity solutions of these systems, only solutions with branes localized in the overall transverse [1] dimensions and those with at most one set of completely localized branes are known [2]–[10]. The cases where all branes are localized have so far eluded solution.

In the past year some progress has been made in finding solutions of partially localized intersecting branes. Some of these results can be found in [2]–[10]. An interesting development has been to interpret brane delocalization physically [11]. In this approach the delocalization seen in the supergravity solution is interpreted via the AdS/CFT correspondence of Maldacena [12, 13] as a Coleman-Mermin-Wagner theorem in the field theory. These results do not directly apply to the case we study and we will see that our near-horizon geometry describes completely localized branes.

In this paper we report an exact solution of 11-dimensional supergravity for a system of intersecting M5-branes in the near-horizon limit. The particular system we study is the supergravity dual of $\mathcal{N} = 2$ superconformal field theory with gauge group SU(N) and $N_f = 2N$ fundamental flavors. This paper is a continuation of our work [6] in which we solved the supersymmetry preservation conditions for the system. The full solution requires solving for a Kähler metric satisfying a non-linear partial differential equation in 7 variables! In our previous paper [6] we solved this equation in an approximation where one set of branes were localized while the second set were smeared out over the worldvolume directions of the first set (these partially localized solutions were also found independently in [5, 7]). This equation was studied in [8] to yield an iterative expansion around the asymptotically flat region. This is the opposite limit to the one we pursue here. In the present paper we solve this differential equation exactly in the near horizon limit which is relevant to the AdS/CFT duality [12].

The paper is structured as follows. We start with a brief description of the system under study. We then take a scaling limit where the Planck scale is taken to infinity while keeping field theory quantities fixed. Finally, we solve for the metric in this "near-horizon" limit, finding a warped AdS geometry. Warped AdS metrics have recently been discussed in the context of the AdS/CFT correspondence and semi-localized intersecting branes [14, 15] for brane configurations similar to ours. We conclude with some comments.

2. The system

In this section we set up the problem and summarize some results from [6] which we will need.

One way of studying $\mathcal{N} = 2$ gauge theories is to generalize the Hanany-Witten [16] set-up to a system relevant to four-dimensional gauge theories [17]. The idea [17] is to suspend D4-branes between a pair of NS5-branes which are separated by a finite coordinate distance L. The gauge theory living on the D4-branes will be, in the infrared, a four-dimensional Yang-Mills theory with $\mathcal{N} = 2$ supersymmetry and gauge group SU(N). There are many ways of introducing fundamental matter, but the easiest method is to introduce semi-infinite D4-branes on either side of the NS5-branes. The gauge D4-branes detect the semi-infinite D4-branes through strings which have ends on both types of D4-branes. These strings carry Chan-Paton factors with respect to the gauge groups of both types of D4-branes. From the gauge theory point of view these represent fundamental matter transforming in the fundamental representation of (a subgroup of) the flavor group.

Witten pointed out that this system can be lifted to M-theory where this web of D4-branes and NS5-branes can be viewed as a single M5-brane wrapping a non-compact Riemann surface which coincides with the Seiberg-Witten Riemann surface. The same picture was derived in a different way in [18].

In the remainder of this paper we will study a configuration of branes consisting of a set of coincident infinite D4-branes intersecting a pair of separated NS5-branes. This configuration can be viewed as one particular realization of SU(N) gauge theory with $N_f = 2N$ according to the recipe described above. Our set-up can be arrived at from any generic Hanany-Witten configuration describing this field theory by moving the D6-branes (on which the semi-infinite D4-branes end "at infinity") through the NS5-branes so that we have an equal number of semi-infinite D4-branes on both sides. We have also tuned the moduli so that all the D4-branes are coincident and collinear. In the gauge theory this corresponds to both tuning the bare masses of the fundamental matter to zero and sitting at the origin of the Coulomb branch where the gauge group is enhanced to the full $\mathrm{SU}(N)$. When viewed from the point of view of M-theory this looks simply like a system of intersecting M5-branes.

It is convenient to pick a coordinate system such that the N D4-branes have world-volume directions along $x^0,\ x^1,\ x^2,\ x^3,\ x^6$ while the NS5-branes have world-volume directions along $x^0,\ x^1,\ x^2,\ x^3,\ x^4,\ x^5$. The two sets of branes then intersect along $x^0,\ x^1,\ x^2,\ x^3$. The positions of the NS5-branes are $x^6 = \pm L/2$. This configuration of branes can be lifted to M-theory with two sets of M5-branes intersecting along $x^0,\ x^1,\ x^2,\ x^3$. Let us denote one set as M5(1) branes, they have world-volume directions along $x^0,\ x^1,\ x^2,\ x^3,\ x^6,\ x^7$ and the other two M5-branes as M5(2), they have world volume directions along $x^0,\ x^1,\ x^2,\ x^3,\ x^4,\ x^5$. The M5(1) branes descend to D4-branes when x^7 is compactified to give type-IIA string theory while the M5(2) branes are localized in the compactified x^7 direction and become NS5-branes. It is convenient to define a complex structure in the subspace $x^4,\ x^5,\ x^6,\ x^7$ as follows:

$$\begin{aligned} v \equiv z^1 &= x^4 + ix^5\,, \\ s \equiv z^2 &= x^6 + ix^7\,. \end{aligned} \tag{2.1}$$

We also take x^7 to be a compact direction with radius R.

In [6] we solved the supersymmetry variation equations for M5-brane configurations which preserve at least 8 real supersymmetries. The general solution is given by the metric:

$$ds^2 = g^{-1/3} dx^2_{3+1} + g^{-1/3} g_{m\overline{n}} dz^m dz^{\overline{n}} + g^{2/3} \delta_{\alpha\beta} dx^\alpha dx^\beta\,, \tag{2.2}$$

and the 4-form field strength:

$$F_{m\overline{n}\alpha\beta} = \frac{i}{4} \epsilon_{\alpha\beta\gamma} \partial_\gamma g_{m\overline{n}}\,, \tag{2.3}$$

$$F_{m89(10)} = -\frac{i}{2} \partial_m g\,, \tag{2.4}$$

$$F_{\overline{m}89(10)} = \frac{i}{2} \partial_{\overline{m}} g\,. \tag{2.5}$$

The Greek indices run over the overall transverse coordinates x^8, x^9, x^{10}. Both the metric and 4-form are expressed in terms of the Kähler metric $g_{m\overline{n}}$. The source equations for the 4-form F force $g_{m\overline{n}}$ to satisfy the non-linear partial differential equations:

$$\partial_\gamma \partial_\gamma g_{m\overline{n}} + 4 \partial_m \partial_{\overline{n}} g = J_{m\overline{n}}\,, \tag{2.6}$$

where J is the source specifying the positions of the M5-branes. The quantity g appearing in the above equations is the square root of the determinant of the Kähler metric: $g = g_{v\overline{v}} g_{s\overline{s}} - g_{v\overline{s}} g_{s\overline{v}}$.

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For the particular configuration that we will be studying the source equations are:

$$
\begin{aligned}
\nabla^2 g_{s\bar{s}} + 4\partial_s\partial_{\bar{s}} g &= -8\pi^3 l_p^3 \delta^{(3)}(r)\left(\delta^{(2)}\left(s-\frac{L}{2}\right)+\delta^{(2)}\left(s+\frac{L}{2}\right)\right),\\
\nabla^2 g_{v\bar{v}} + 4\partial_v\partial_{\bar{v}} g &= -N8\pi^3 l_p^3 \delta^{(3)}(r)\delta^{(2)}(v)\,, \qquad \nabla^2 g_{v\bar{s}} + 4\partial_v\partial_{\bar{s}} g = 0\,,
\end{aligned} \tag{2.7}
$$

where ∇^2 is the flat laplacian in the overall transverse space.

To summarize, a given M5-brane configuration determines a source J in (2.6). Solving this source equation for the metric $g_{m\bar{n}}$ then determines all other quantities in the supergravity solution.

3. Field theory ("near-horizon") limit

Maldacena proposed a certain scaling limit of string theory quantities [19, 12] to isolate the world-volume gauge theory from bulk interactions. The idea is simply to take a limit in which the Planck length goes to zero while keeping field theory quantities fixed. In [6] we pointed out the relevant scalings of supergravity variables in type-IIA theory. We can express these scalings in M-theory units by defining w, t and y as follows:

$$
w = \frac{v}{\alpha'} = \frac{vR}{l_p^3}\,, \qquad t^2 = \frac{r}{g_s\alpha'^{3/2}} = \frac{r}{l_p^3}\,, \qquad y = \frac{s}{R}\,. \tag{3.1}
$$

Note that w and y are complex variables while t is a real variable. The field theory limit is one in which we keep w, y, t fixed while taking l_p to zero. We take the metric to be:

$$
\frac{1}{l_p^2}ds^2 = g^{-1/3}\eta_{\mu\nu}dx^\mu dx^\nu + g^{-1/3}g_{m\bar{n}}dz^m dz^{\bar{n}} + g^{2/3}\left(4t^2dt^2 + t^4 d\Omega_2^2\right), \tag{3.2}
$$

where now m, n run over y, w, $g = g_{w\bar{w}}g_{y\bar{y}} - g_{w\bar{y}}g_{y\bar{w}}$ and $d\Omega_2^2$ is the metric on the round unit 2-sphere. The source equations become:

$$
\begin{aligned}
\frac{1}{4t^5}\partial_t(t^3\partial_t)g_{y\bar{y}} + 4\partial_y\partial_{\bar{y}} g &= -\pi^2\frac{\delta(t)}{t^5}\left(\delta^{(2)}\left(y-\frac{1}{2g_{\rm YM}^2}\right)+\delta^{(2)}\left(y+\frac{1}{2g_{\rm YM}^2}\right)\right),\\
\frac{1}{4t^5}\partial_t(t^3\partial_t)g_{w\bar{w}} + 4\partial_w\partial_{\bar{w}} g &= -N\pi^2\frac{\delta(t)}{t^5}\delta^{(2)}(w)\,,\\
\frac{1}{4t^5}\partial_t(t^3\partial_t)g_{w\bar{y}} + 4\partial_w\partial_{\bar{y}} g &= 0\,,
\end{aligned} \tag{3.3}
$$

where $g_{\rm YM}^2 = R/L$ is the Yang-Mills coupling constant in the field theory. We have also assumed that the metric depends only on the "radial" coordinate t and not the "angular" variables in the overall transverse directions. This is consistent with the requirement of having an SU(2) isometry corresponding to the field theory R-symmetry.

We end this section with some comments. The source equations now have no powers of the Planck length l_p, they are expressed in terms of quantities which have a field theory interpretation. This will allow us to express the metric in terms of only these rescaled variables with no further dependence on l_p aside from the overall multiplicative factor. Secondly, we would like to point out that while the initial set up treated the M5(1) and M5(2) branes on an equal footing, the scaling limit we take breaks that symmetry. This is clear from the type-IIA picture since there the D4-branes play a distinguished role in that the field theory of interest lives on their world-volume.

4. Near-horizon geometry of intersecting M5-branes

Since we are looking for a supergravity dual of a four-dimensional conformal field theory, we expect a solution where the metric contains an AdS_5 factor. The most general metric of this type is:

$$\frac{1}{l_P^2}ds^2 = \Omega^2\left(u^2 dx_{3+1}^2 + \frac{1}{u^2}du^2\right) + ds_6^2\,, \tag{4.1}$$

where the metric for the six-dimensional transverse space and Ω are independent of the AdS_5 coordinates.

It is convenient to choose variables where there is only one-dimensionful variable. We choose to do this by defining ρ with dimensions of mass and dimensionless angular variables θ and ϕ by:

$$\begin{aligned} t &= \rho\cos\theta\,,\\ w &= \rho\sin\theta e^{i\phi}\,. \end{aligned} \tag{4.2}$$

Now we see on dimensional grounds that u must be related to ρ by $u = \rho\alpha$ where α is some function of the dimensionless variables θ, ϕ, y and $\bar{y}$. By substituting this expression for u into the above metric, we can compare the metric components with those in the known form of the solution, eq. (3.2). In particular, by examining the factor multiplying dx_{3+1}^2 and the metric components $g_{\rho\rho}$, $g_{\rho y}$, $g_{\rho\theta}$ and $g_{\rho\phi}$ we find:

$$\begin{aligned} g &= \frac{1}{\Omega^6\alpha^6\rho^6}\,, \qquad g_{w\bar{w}} = \frac{\Omega^6\alpha^4 - 4\cos^4\theta}{\rho^4\Omega^6\alpha^6\sin^2\theta}\,, \qquad g_{y\bar{w}} = \frac{2e^{i\phi}\partial_y\alpha}{\rho^3\alpha^3\sin\theta}\,,\\ \partial_\theta\alpha &= \frac{(\Omega^6\alpha^4 - 4\cos^2\theta)\cos\theta}{\Omega^6\alpha^3\sin\theta}\,, \qquad \partial_\phi\alpha = 0\,. \end{aligned} \tag{4.3}$$

Since we are looking at $\mathcal{N} = 2$ superconformal field theories we would like to preserve a $\mathrm{SU}(2)\times\mathrm{U}(1)$ isometry. This we have incorporated in the above ansatz by requiring that the metric preserve a U(1) which rotates w by a phase and the SU(2) symmetry of the transverse 2-sphere. In fact, these symmetries are consequences of our required form of the metric. For example we see that α is independent of ϕ and the equation for $\partial_\theta\alpha$ shows that Ω must also be independent of ϕ.

If we assume, for the moment, that Ω is constant,[1] say Ω_0, we can solve for the θ-dependence of α (which we denote by α_0) in terms of an arbitrary function $A(y,\bar{y})$:

$$\Omega_0^6\alpha_0^4 = 4\cos^4\theta + 4A(y,\bar{y})\sin^4\theta\,. \tag{4.4}$$

We can then write the above equations as:

$$g = \frac{\Omega_0^3}{8\rho^6\beta^3}\,, \tag{4.5}$$

$$g_{w\bar{w}} = \frac{\Omega_0^3 A\sin^2\theta}{2\rho^4\beta^3}\,, \tag{4.6}$$

$$g_{y\bar{w}} = \frac{e^{i\phi}\Omega_0^3\sin^3\theta\partial_y A}{4\rho^3\beta^3}\,, \tag{4.7}$$

where we have defined:

$$\beta = \left(\cos^4\theta + A\sin^4\theta\right)^{1/2}. \tag{4.8}$$

The metric component $g_{y\bar{y}}$ can be determined from the determinant g and the other components of the metric given in the above equations. However, the metric determined in this way fails to be Kähler.[2] It is easier instead to determine $g_{y\bar{y}}$ by requiring the metric to be Kähler. The Kähler condition is satisfied if:

$$g_{y\bar{y}} = \frac{\Omega_0^3\sin^4\theta|\partial_y A|^2}{8\rho^2 A\beta^3} \tag{4.9}$$

provided that $A = |F(y)|^2$, where F is a holomorphic function of y. One can easily check that the source equations are satisfied everywhere away from the support of the delta functions.[3] The metric as it stands now has a vanishing determinant so it is not a valid solution. However, it is easy to see that the metric can be modified in such a way as to get the correct determinant while continuing to satisfy the source equations. The idea is simply to add to all the metric components additional terms which are themselves Kähler (so as not to destroy the Kähler properties of our initial ansatz), such that the determinant is correctly reproduced. The source equations

[1]As we will see below the metric has a warped AdS structure and so our assumption that Ω is constant is incorrect. Nevertheless, for the purposes of solving the equations we find it easier to begin with an incorrect assumption which can be easily modified to yield the correct metric than to solve the equations directly.

[2]Hence the assumption of constant Ω is incorrect.

[3]The normalizations and precise form of F relevant to the delta function sources are determined below.

will continue to be satisfied provided these additional terms do not depend on t. From these simple requirements one determines the solution:

$$g_{w\bar{w}} = \frac{\Omega_0^3 A \sin^2\theta}{2\rho^4\beta^3} + \frac{|f|^2}{\rho^4\sin^4\theta}\,, \qquad g_{y\bar{y}} = \frac{\Omega_0^3\sin^4\theta|\partial_y A|^2}{8\rho^2 A\beta^3} + \frac{|\partial_y f|^2}{\rho^2\sin^2\theta}\,,$$
$$g_{y\bar{w}} = \frac{e^{i\phi}\Omega_0^3\sin^3\theta\partial_y A}{4\rho^3\beta^3} - \frac{e^{i\phi}\bar{f}\partial_y f}{\rho^3\sin^3\theta}\,, \tag{4.10}$$

where $f(y)$ is a holomorphic function determined from the requirement that the determinant has the form (4.5). This requirement can be stated succinctly as a differential equation:

$$\partial_y(f^2 F) = f\,. \tag{4.11}$$

The general solution of this equation is:

$$f(y) = \frac{\int_a^y F^{-1/2}(z)dz}{2F^{1/2}(y)}\,, \tag{4.12}$$

where a is an arbitrary constant of integration.

Note that the additional terms in the metric (involving f) are independent of t (r in the original coordinates) while the other terms in $g_{m\bar{n}}$ can be expressed in the form $g\partial_m\partial_{\bar{n}}(|Fw^2|^2)$. So the source equations can be conveniently written as:

$$\frac{1}{4t^5}\partial_t(t^3\partial_t g)\partial_m\partial_{\bar{n}}\left(|Fw^2|^2\right) + 4\partial_m\partial_{\bar{n}} g = J_{m\bar{n}}\,. \tag{4.13}$$

It is now a straightforward calculation to check that these equations are satisfied with the source terms:

$$J_{w\bar{w}} = -\pi^2 N\frac{\delta(t)}{t^5}\delta^{(2)}(w)\,,$$
$$J_{y\bar{y}} = -\pi^2\frac{\delta(t)}{t^5}\left(\delta^{(2)}\left(y - \frac{1}{2g_{\rm YM}^2}\right) + \delta^{(2)}\left(y + \frac{1}{2g_{\rm YM}^2}\right)\right),$$
$$J_{y\bar{w}} = 0\,. \tag{4.14}$$

The source equations determine F (and consequently also f) since the M5(2) branes are localized at the zeroes of F, as well as fixing the constant:

$$\Omega_0^3 = 4\pi N\,. \tag{4.15}$$

We will now give the explicit form of the metric, exhibiting the warped product structure before solving for F. We will then consider the form of F in the large radius limit before solving it in the general case.

4.1 Warped Anti-de Sitter structure of the metric

According to Maldacena's conjecture [12] conformal field theories have Anti-de Sitter supergravity duals. Our metric is not a product manifold of Anti-de Sitter space with a transverse manifold, but as mentioned earlier in this section, the metric can be written as a warped product consistent with Maldacena's conjecture. To see this warped product structure, one simply returns to eqs. (4.3) and solves for α and Ω using the explicit metric appearing in (4.10). This yields:

$$\begin{aligned}\alpha^{-2} &= \frac{2\pi N}{(\cos^4\theta + |F|^2\sin^4\theta)^{1/2}} + \frac{|f|^2}{\sin^2\theta}\,,\\ (\Omega\alpha)^{-6} &= \frac{\pi N}{2}\frac{1}{(\cos^4\theta + |F|^2\sin^4\theta)^{3/2}}\,.\end{aligned} \tag{4.16}$$

These expressions are consistent with all the metric components derived and it can easily be checked that $\partial_\theta\alpha$ has the correct form required by eq. (4.3). The metric, therefore, can be written as a warped product of AdS space with a transverse manifold. The metric, while messy, can be written relatively concisely if one expresses it in terms of α and Ω:

$$\begin{aligned}\frac{1}{l_P^2}ds^2 = {}& \Omega^2\left(u^2\eta_{\mu\nu}dx^\mu dx^\nu + \frac{du^2}{u^2}\right) + \frac{4\cos^2\theta}{\Omega^4\alpha^4\sin^2\theta}\left(1 - \frac{4\cos^4\theta}{\Omega^6\alpha^4}\right)d\theta^2 + \\ &+ \frac{8\cos^3\theta}{\sin\theta\Omega^4\alpha^5}\partial_y\alpha d\theta dy + \frac{8\cos^3\theta}{\sin\theta\Omega^4\alpha^5}\partial_{\bar y}\alpha d\theta d\bar y + \\ &+ \Omega^2\left(1 - \frac{4\cos^4\theta}{\Omega^6\alpha^4}\right)d\phi^2 - 2i\Omega^2\frac{\partial_y\alpha}{\alpha}d\phi dy + 2i\Omega^2\frac{\partial_{\bar y}\alpha}{\alpha}d\phi d\bar y + \\ &+ \frac{\Omega^2\alpha^2}{\Omega^6\alpha^4 - 4\cos^4\theta}\left(\sin^2\theta + \left(2\Omega^6\alpha^4 + 8\cos^4\theta\right)\left|\frac{\partial_{\bar y}\alpha}{\alpha^2}\right|^2\right)||dy|^2 - \\ &- \frac{\Omega^2}{\alpha^2}(\partial_y\alpha)^2dy^2 - \frac{\Omega^2}{\alpha^2}(\partial_{\bar y}\alpha)^2d\bar y^2 + \frac{\cos^4\theta}{\Omega^4\alpha^4}d\Omega_2^2\,.\end{aligned} \tag{4.17}$$

Everything is now determined explicitly in terms of $F(y)$. We will now consider the form of F in various cases, including the simple generalisation to conformal theories with gauge group $\mathrm{SU}(N)^n$.

4.2 Large R or M-theory limit

In the limit that R, the radius of x^7, becomes infinite we can ignore the periodicity of y: $y \to y + i2\pi$. Notice that the field theory is not sensitive to the value of R but only to the ratio R/L which determines the gauge coupling constant. We are thus simultaneously taking L to infinity while keeping R/L fixed. In this limit we can solve for F taking into account the normalization of the sources:

$$F = \left(\left(y - \frac{1}{2g_{\mathrm{YM}}^2}\right)\left(y + \frac{1}{2g_{\mathrm{YM}}^2}\right)\right)^{2/N}\,. \tag{4.18}$$

Using this explicit expression we can calculate f:

$$f=\frac{1}{2}(4g_{\rm YM}^{4})^{1/N}y\left(y^{2}-\frac{1}{4g_{\rm YM}^{4}}\right)^{-1/N}\mathcal{F}\left(\frac{1}{N},\frac{1}{2};\frac{3}{2};4g_{\rm YM}^{4}y^{2}\right), \tag{4.19}$$

where $\mathcal{F}$ denotes the hypergeometric function.

It is easy to see how one can generalize this to an arbitrary number of M5(2) branes. If there are n M5(2) branes located at $y=y_i$ then:

$$F=\prod_{i=1}^{n}(y-y_i)^{2/N}\,. \tag{4.20}$$

We can then determine, at least in principle, f from this expression. The dual conformal field theory will have a product gauge group $\mathrm{SU}(N)^{n-1}$ with the gauge coupling of the i'th factor being given by:

$$\frac{1}{g_{{\rm YM},i}^{2}}=y_{i+1}-y_i\,. \tag{4.21}$$

4.3 Solution for arbitrary R

As noted above the zeroes of F determine the locations of the M5(2) branes. From the previous section it is easy to see how to generalize to an arbitrary radius of x^7 (i.e. when we take into account the periodicity of y). For our sources with periodic y the correct F is:

$$F=\left(\sinh\left(y-\frac{1}{2g_{\rm YM}^{2}}\right)\sinh\left(y+\frac{1}{2g_{\rm YM}^{2}}\right)\right)^{2/N}. \tag{4.22}$$

In this case we have not been able to express f in terms of a known function but it is still given by the integral in (4.12). One can similarly generalize this for a collection of n M5(2) branes:

$$F=\prod_{i=1}^{n}\sinh(y-y_i)^{2/N}\,. \tag{4.23}$$

This again determines f in principle through eq. (4.12).

5. Conclusions and discussion

In this paper we presented an exact solution of 11-dimensional supergravity describing localized intersections of M5-branes. The solution has some surprising features worth pointing out.

The geometry of our intersecting brane configuration is a warped Anti-de Sitter product geometry, consistent with the fact that the dual quantum field theory is a conformal field theory.

Another feature concerns the ‘t Hooft coupling. Taking the large-N limit to remain in the domain of validity of supergravity does not imply anything about the value of $g_{\rm YM}^2 = R/L$. This ratio is an arbitrary constant. In the solutions known thus far for 4-dimensional field theories the relevant combination appearing in the supergravity solution is always $g_{\rm YM}^2 N$, forcing the ‘t Hooft coupling to be large in the small curvature limit relevant to supergravity. In our case there appears to be no such restriction on the ‘t Hooft coupling. It is thus surprising that in principle one can tune the ‘t Hooft coupling to be small or large while remaining in the domain of validity of supergravity. However, the large-N limit may be rather subtle in this case and this issue is currently under investigation.

Our solution does not have any simple-N dependence: there are terms of different orders in N in a $1/N$ expansion despite the fact that we have taken the decoupling limit. Unlike the $AdS_5 \times S^5$ case, the $1/N$ suppressed terms do not come with powers of the Planck scale. Certainly further terms relevant to the asymptotically flat solution will contain the Planck scale, however, it is surprising that the $1/N$ corrections do not appear to be directly connected to an expansion in the Planck scale.

There are a number of directions which open up from this analysis. One is to consider other intersecting branes which are connected to this configuration through compactification and T-duality. The present system of M5-branes can be viewed as a special case of a more general problem of an M5-brane wrapped on a Riemann surface. The supergravity description of the general problem will be of interest for finding supergravity duals for more interesting field theories, including non-conformal field theories. The solution to the latter problem will be presented in [20].

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References

[1] J.P. Gauntlett, *Intersecting branes*, `hep-th/9705011`.

[2] N. Itzhaki, A.A. Tseytlin and S. Yankielowicz, *Supergravity solutions for branes localized within branes*, *Phys. Lett.* **B 432** (1998) 298 [`hep-th/9803103`].

[3] A. Hashimoto, *Supergravity solutions for localized intersections of branes*, *J. High Energy Phys.* **01** (1999) 018 [`hep-th/9812159`].

[4] H.S. Yang, *Localized intersecting brane solutions of $D = 11$ supergravity*, hep-th/9902128.

[5] D. Youm, *Localized intersecting BPS branes*, hep-th/9902208.

[6] A. Fayyazuddin and D.J. Smith, *Localized intersections of M5-branes and four-dimensional superconformal field theories*, *J. High Energy Phys.* **04** (1999) 030 [hep-th/9902210].

[7] A. Loewy, *Semi localized brane intersections in SUGRA*, *Phys. Lett.* **B 463** (1999) 41 [hep-th/9903038].

[8] A. Gomberoff, D. Kastor, D. Marolf and J. Traschen, *Fully localized brane intersections: the plot thickens*, *Phys. Rev.* **D 61** (2000) 024012 [hep-th/9905094].

[9] D. Youm, *Supergravity solutions for bi-dyons*, *Phys. Rev.* **D 60** (1999) 105006 [hep-th/9905155].

[10] S.A. Cherkis, *Supergravity solution for M5-brane intersection*, hep-th/9906203.

[11] D. Marolf and A. Peet, *Brane baldness vs. superselection sectors*, *Phys. Rev.* **D 60** (1999) 105007 [hep-th/9903213];
A.W. Peet, *Baldness/delocalization in intersecting brane systems*, *Class. and Quant. Grav.* **17** (2000) 1235 [hep-th/9910098].

[12] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[13] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[14] M. Alishahiha and Y. Oz, *AdS/CFT and BPS strings in four dimensions*, *Phys. Lett.* **B 465** (1999) 136 [hep-th/9907206];
Y. Oz, *Warped compactifications and AdS/CFT*, hep-th/0004009.

[15] M. Cvetič, H. Lu, C.N. Pope and J.F. Vazquez-Poritz, *AdS in warped spacetimes*, hep-th/0005246.

[16] A. Hanany and E. Witten, *Type-IIB superstrings, BPS monopoles and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[17] E. Witten, *Solutions of four-dimensional field theories via M-theory*, *Nucl. Phys.* **B 500** (1997) 3 [hep-th/9703166].

[18] A. Klemm, W. Lerche, P. Mayr, C. Vafa and N. Warner, *Self-dual strings and $N = 2$ supersymmetric field theory*, *Nucl. Phys.* **B 477** (1996) 746 [hep-th/9604034].

[19] J.M. Maldacena *Branes probing black holes*, *Nucl. Phys.* **68** *(Proc. Suppl.)* (1998) 17 [hep-th/9709099].

[20] B. Brinne, A. Fayyazuddin, S. Mukhopadhyay and D.J. Smith, in preparation.

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